Homework 3, due Thursday June 30th

In this homework you will solve first order linear and Bernoulli equations using integrating factors.

1) Write the following differential equations in the form $y' + p(x)y = q(x)$ and then write the corresponding integrating factors. DO NOT TRY TO SOLVE!

a) $y' = 10 - 15y$

b) $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1$

c) $xy' - 2y = x^5 \sin(2x) - x^3 + 4x^4$

2) Consider the differential equation given by:

$$y' + \frac{4}{x}y = x^4$$

a) Write this in the form $y' + p(x)y = q(x)$. Find the appropriate integrating factor.

b) Calculate $\frac{d}{dx}(e^{\int p(x)dx}y)$ using the product rule.

c) Write the result of multiplying both sides of the equation from part a) by the integrating factor.

Notice that the left side of the equation in part c) is the same as the calculation in part b). We are just doing the product rule backwards! So in general when we multiply by the integrating factor this will always work, you just use the product rule to change the left side of the equation into the much simpler version which is always:

$$\frac{d}{dx}(e^{\int p(x)dx}y)$$

d) After rewriting the left side of c) as $\frac{d}{dx}(e^{\int p(x)dx}y)$ solve the differential equation by integrating both sides with respect to x.

3) Consider the differential equation

$$y' - \sin(x)y = 0$$

a) Solve this by using an integrating factor.

b) Solve this by using separation of variables.

Math is consistent. If you solve the same problem in two different yet still correct ways you will get the same answer.

4) Consider the differential equation:

$$y' - \frac{3}{4}y = x^4 y^{\frac{1}{3}}$$

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a) This equation is Bernoulli- identify $p(x), q(x)$ and n.

b) Then write out the substitution $z = y^{1-n}$. Using this you should also get that $y = z^{3/2}$. Find y' in terms of z and z'.

c) Write out the result of substituting y and y' with the equations in b).

d) This equation is now linear in terms of z. Solve it using an integrating factor.

e) You now have a formula for z, plug this back into $z = y^{1-n}$ and solve for y to finish the problem.

In general we do this exact same procedure for any Bernoulli equation. We make a substitution, which makes it linear, which we can solve, which we then resubstitute to get the final answer.

Practice Problems - Do not turn these in

Doing the following problems will benefit you. Practice makes perfect and **math is not a spectator sport**.

P1) Solve $y' + y = \sin(x)$ such that $y(\pi) = 1$. (Hint: You will need integration by parts when you integrate the right hand side after multiplying by the integrating factor.)

P2) Solve $\frac{dz}{dx} - xz = -x$ such that $z(0) = -4$.

P3) Solve $y' = \cos(x)y^4$ using separation of variables and then by using the fact that it is Bernoulli with $n = 4$. Make sure you get the same answer in both cases.

P4) Solve $y' + 3x^2y = 0$.

P5) Solve $xy' + y = xy^3$.

P6) Solve $y' - 7y = e^x$.

P7) Solve $y' + y = y^2e^x$.